Intermediate Mathematics

Outline for INTERMEDIATE MATHEMATICS

- Law of Sines
- Law of Cosines
- Linear Equations
  - Slope-Intercept Method
  - Point-Point Method
  - Point-Slope Method
- Parallel & Perpendicular Lines
- Points of Intersections of Two Lines
- Vectors
- Quadratic Formula
- Mixtures
- Relationship Between Numbers (Story Problems)

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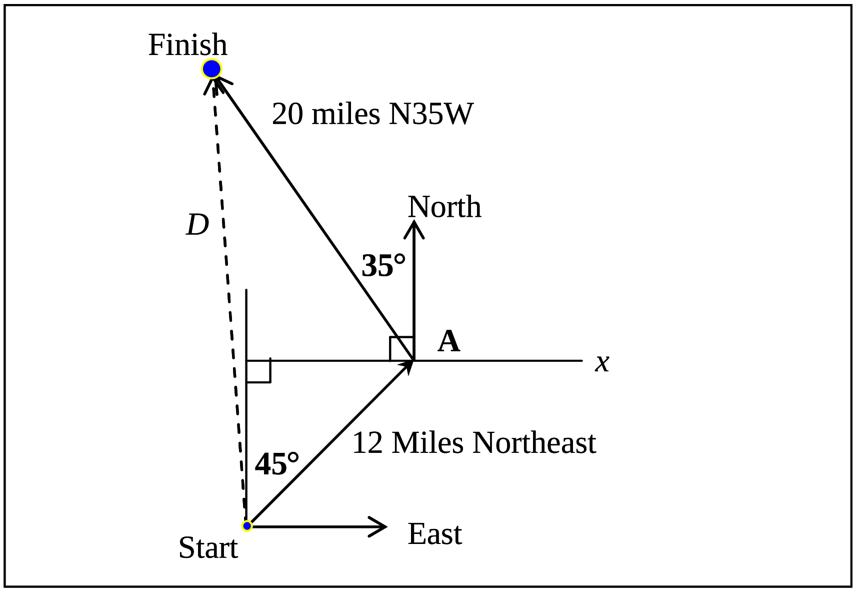
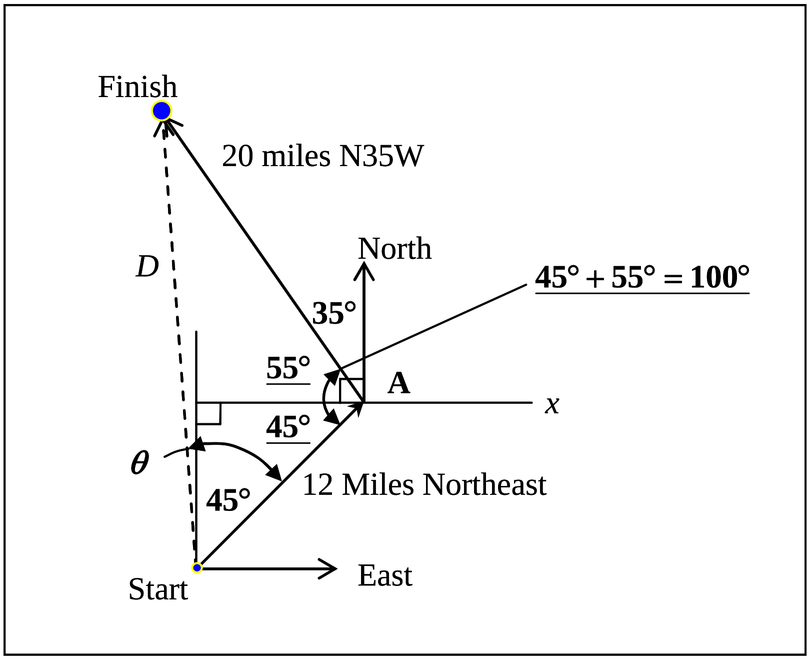
Problem #9: Find the shortest distance and bearing between our start and finish positions if our journey had taken us northeast for 3 hours (45 degrees if noted without degree) and then we followed a bearing of N35°W for a period of 5 hours.

Solution #9: Let's start by drawing a diagram to represent what is physically occurring.



We know that the direction northeast defines a 45° angle. We also know, as shown below, that the next angle up on our diagram is 45° (shown underlined in the following diagram) because it is an interior angle between two parallel lines. The angle above that one has to be 55° (also shown underlined in the following diagram) because we know that there are 90° in a right angle and 35 of those degrees are used up by our North 35° West bearing. It is now easy to determine that the angle in our triangle, at point "A", is 100°. That is the sum of 45° and 55°. With that said, where do we go from here?



Actually, there are several ways to solve this problem, but we are, at this point, limited to trigonometry. And, in fact, that's not a bad way to solve this problem. Again, our trig skills tell us that we can't use the Pythagorean Theorem or the trig functions for this triangle because it is not a right triangle. Our only other two choices are the Law of Sines and the Law of Cosines.

When we look this diagram over, we see, unfortunately, that we don't have values for any given angle and its opposite side. We know that the angle at "A" is 100 degrees, but we don't know side (D). We know the 20 mile side, but we don't know the angle θ. And, lastly, we know the 12 mile side, but we don't know the angle opposite from that side either. As we said a minute ago, this is unfortunate because we must now eliminate the Law of Sines.

By the very process of eliminating all the fun, (easy possibilities in terms of solving this problem), it looks like we're stuck with that old clunker which we call the Law of Cosines.

Anyway, as despicable as it may be, we might as well "bite the bullet", so to speak, and get on with it.

If you will recall our discussion of the Law of Sines, we said that it can be used when we know any two sides of a triangle and the angle included between those sides. In this case, that is exactly what we know. Specifically, we know the 12 mile side, the 20 mile side, and the angle which is included between them, namely; the 100 degree angle.

You will also recall that the Law of Cosines states that: "the square of any side of a triangle is equal to the sum of the squares of the other two sides, minus twice their product multiplied by the cosine of the angle opposite to the side in question" or

$$c^2 = a^2 + b^2 - 2ab\cos\theta_c$$

You may wish to take a moment or two to review that definition. Anyway, here is what we have in terms of an equation:

$$D^2 = 20^2 + 12^2 - 2(20)(12)\cos(100)$$

If we multiply these numbers out, we find that:

$$D^2 = 400 + 144 - 2(20)(12)\cos(100)$$

Since the cosine of 100 degrees is -0.1736, this last string of numbers multiplies out to plus 83.35. Adding all this up, our equation becomes:

$$D^2 = 544 - (-83.35) = 544 + 83.35$$

or approximately

$$D^2 = 627$$

When we take the square root of 627, we find that

$$\underline{D = \text{about } 25\text{ miles}}$$

Good job! We're almost there.

The next thing that we have to do is find our angle, θ. That will allow us to get our bearing, so to speak. Well, finding θ should be easy because we now know *D*. Therefore, we can use the Law of Sines. Stick around, it gets better.

The LAW OF SINES states:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The Law of Sines allows us to write the following equation, recalling $D = 25\,\text{miles}$:

$$\frac{25}{\sin 100^\circ} = \frac{20}{\sin\theta}$$

We find on the calculator the $\sin 100^\circ = 0.985$ so:

$$\frac{25}{0.985} = \frac{20}{\sin\theta}$$

If we multiply both sides of our equation by both the $\sin\theta$ and 0.985, we get:

$$25\sin\theta = (0.985)(20)$$

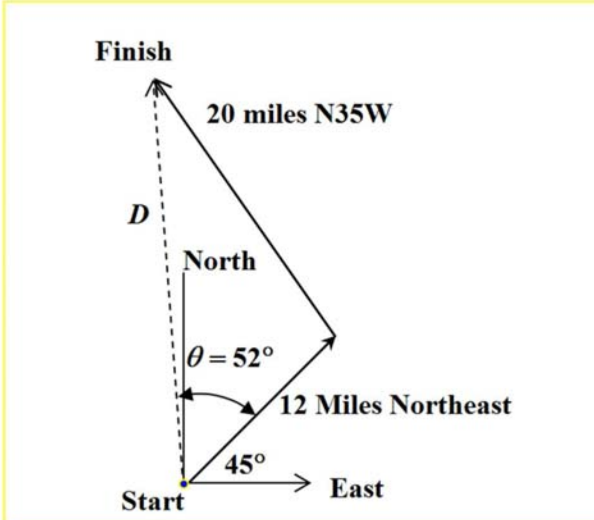
To arrive at the sin of θ, we just divide both sides of our equation by 25.

$$\sin\theta = (0.985)(20)/25 = 0.788$$

To find the inverse sine, we can now just put the 0.788 on the display of our calculator, hit the 2nd button, followed by the sine button and we find

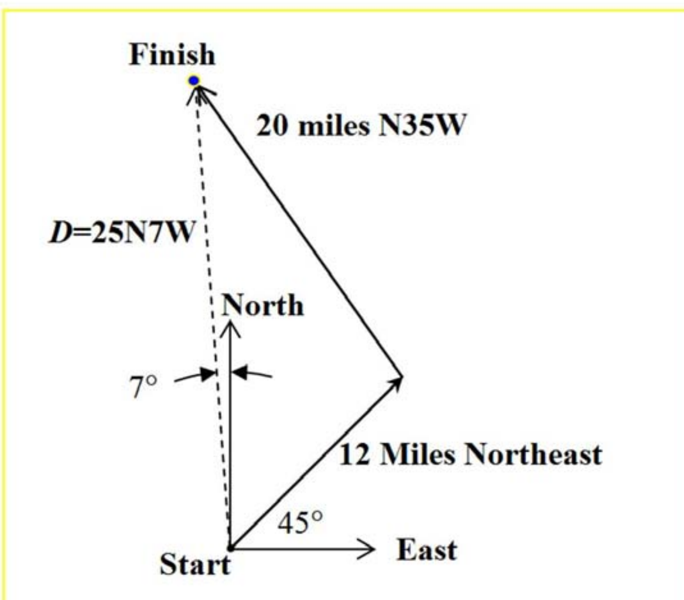
$$\underline{\theta = 52\text{ degrees}}$$

Adding the 52 degrees to the 45 degrees at our start point, we see that the total angle which vector D makes with the horizontal axis is 97 degrees.



However, if we are going to determine our bearing, we are going to have to subtract 90 degrees from the 97 degrees. That is because we have to rotate through 90 degrees to get ourselves up to the north, or vertical, bearing line. The 7 degrees which remains is, in fact, our bearing value. With this subtraction, we started with due east and rotated 97 degrees counterclockwise. That brought us to 7 degrees past due north. Therefore, our final bearing is north, 7 degrees west, or N7W.

Our bottom line answer, then, is 25 miles at a bearing of north, 7 degrees west, or N7W as shown in the diagram below.



Wasn't that a great problem? Aside from all the profanities and stuff, we think we did a very fine job. It used a number of the tools which we covered previously and forced us to understand the problem so that we could choose the proper solution technique. If you followed this analysis at all, which we know you did, the vector portion of this work element is in your back pocket.

Realistically, this is not quite rocket science. But, on the other hand, it is well beyond the everyday mathematics which the vast majority of people deal with on a daily basis. If you have persevered to this point, you can be extremely proud of yourself.